MAE 166A : Analysis of Flight Structures : Assignment 7

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PROBLEM 4.11

We're given a 2 meter long, thin-walled beam, fixed at one end, with section identical to the one from a previous assignment, Problem 4.4, Section 4.21. The height of the section is given as, $2\ h = 0.4$ m, the thickness, $t = 2.0\times 10^{-3}$ m, and the cross-sectional area of the stringers, $A' = 2.5\times 10^{-3}$ m^2. We'll define our coordinate axes with origin located at the centroid of the beam section at the fixed end of the beam. Referring back to Problem 4.4 on the previous assignment, the moments of inertia for the beam may be written as

$$I_y = 4\ h^2\ A' = 4.0 \times 10^{-4}\ \text{m}^4, \tag{1}$$

$$I_z = h^2\ A' = 1.0 \times 10^{-4}\ \text{m}^4, \tag{2}$$

$$I_{yz} = 0. \tag{3}$$

A shear force, $V_z = 5000$ N, acts on the beam's free end. The Temoshenko equations give rise to the following two definitions,

$$\psi_y = \frac{V_z}{2\ E\ I_y}\ x^2 + B_1\ x + B_0, \tag{4}$$

$$w_0 = \frac{V_z}{G\ A}\ x - \frac{V_z}{6\ E\ I_y}\ x^3 - \frac{B_1}{2}\ x^2 - B_0\ x + C_0, \tag{5}$$

where B_1, B_0, and C_0 are constants of integration and $A = 4\ A' = 1.0 \times 10^{-2}$ m^2 represents the total cross-sectional area of the beam. Boundary conditions require the deflection, w_0, of the beam at the fixed end to go to zero, allowing us to write

$$w_0(x = 0) = 0 = C_0. \tag{6}$$

The cross-sectional rotation, ψ_y, of the beam must also be zero at the fixed end, giving us

$$\psi_y(x = 0) = 0 = B_0. \tag{7}$$

Since the force is applied at the free end, the moment, M_y, must be zero there, such that

$$M_y(x = 2) = \frac{d^2 w_0}{dx^2}(x = 2) = 0 = -\frac{2\ V_z}{E\ I_y} - B_1. \tag{8}$$

Solving for the constant B_1 gives us

$$B_1 = -\frac{2\ V_z}{E\ I_y}. \tag{9}$$

Substituting these constant back into our deflection equation, we can write

$$w_0 = \frac{V_z}{G\ A}\ x - \frac{V_z}{6\ E\ I_y}\ x^3 + \frac{V_z}{E\ I_y}\ x^2. \tag{10}$$

For a beam composed of aluminum 2024-T3, the shear modulus is given as, $G = 27$ GPa, and Young's modulus is $E = 70$ GPa. Substituting in our transverse shear force, $V_z = 5000$ N and our moment of inertia, $I_y = 4.0 \times 10^{-4}$ m^4 gives us

$$w_0 = (1.85\times 10^{-5})\ x - (2.98\times 10^{-5})\ x^3 + (1.79\times 10^{-4})\ x^2. \tag{11}$$

The total deflection experienced by the end of the beam then is given by

$$w_0(x = 2) = 5.13 \times 10^{-4}\ \text{m}. \tag{12}$$

PROBLEM 5.3

(a)

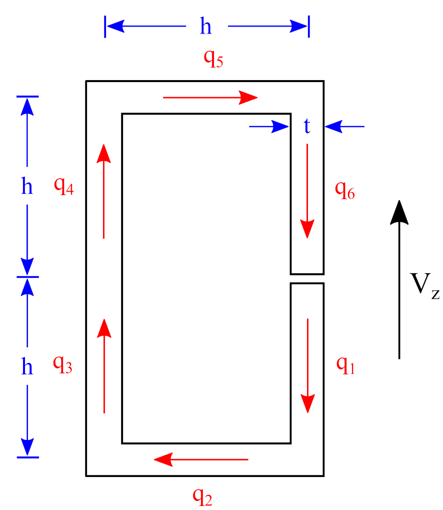

FIG. 1: 5.1 : Thin-walled section with side cut showing flexural shear flow components.

We're given a thin-walled beam with section as shown in Figure 1, where $h = 0.1$ m and $t = 2.0 \times 10^{-3}$ m. From a previous assignment, Problem 5.1, The shear flow through the section was calculated for a beam subject to a transverse shear force, $V_z = 1.0 \times 10^3$ N such that

$$q_1 = q_6 = -(1.5 \times 10^5)\ s^2,$$

$$q_2 = q_5 = -(1.5 \times 10^3) - (3.0 \times 10^4)\ s,$$

$$q_3 = q_4 = -(4.5 \times 10^3) - (3.0 \times 10^4)\ s + (1.5 \times 10^5)\ s^2.$$

We can integrate each of these to get the total shear acting in each section,

$$V_1 = V_6 = \int_0^h q_1\ ds = -(0.5 \times 10^5)\ h^3 = -50.0 \text{ N},$$

$$V_2 = V_5 = -300.0 \text{ N},$$

$$V_3 = V_4 = -550.0 \text{ N}.$$

Given these shears, we can determine the location of the shear center for the cross-section, y_{sc}, through the relation,

$$V_z\ y_{sc} = 2\left[V_1\ (h/2) + V_2\ (h) + V_3\ (h/2)\right]. \tag{13}$$

Solving for y_{sc}, the horizontal displacement of the shear center from the centroid (the vertical location being equivalent to that of the centroid due to symmetry), we get

$$y_{sc} = -0.12 \text{ m}. \tag{14}$$

(b)

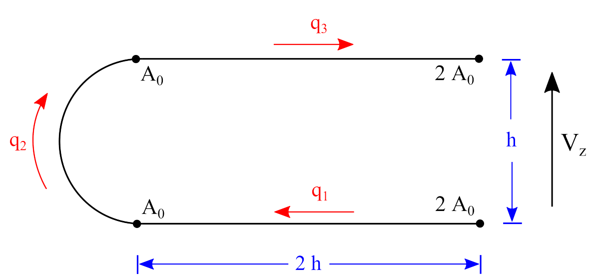

FIG. 2: 5.4 : Open, four-stringer section showing flexural shear flow components.

We're given a stringer and web section as shown in Figure 2. From a previous assignment, Problem 5.4, The shear flow through the section was calculated for a beam subject to a transverse shear force, $V_z = 5.0 \times 10^3$ N such that

$$q_1 = -3.33 \times 10^3 \text{ N m}^{-1},$$

$$q_2 = -5.00 \times 10^3 \text{ N m}^{-1},$$

$$q_3 = -3.33 \times 10^3 \text{ N m}^{-1}. \tag{15}$$

Given these shear flows, we can determine the location of the shear center for the cross-section, y_{sc}, through the relation,

$$V_z\ y_{sc} = q_1\ (2\ h)\ (h/2) + Q_2 + q_3\ (2\ h)\ (h/2), \tag{16}$$

where Q_2 is the component of torque resulting from the shear flow, q_2. From the geometry of the section, we can write this as

$$Q_2 = \int_{-\pi/2}^{\pi/2} q_2 \left(r + \frac{4}{3}\ h\ \cos(\theta)\right) r\ d\theta,$$

$$= q_2\ r\left(\pi\ r + \frac{8}{3}\ h\right), \tag{17}$$

where $r = h/2$ represents the turning radius of the second web section. In terms of this radius, we can rewrite Q_2 as

$$Q_2 = q_2\ h^2\left(\frac{\pi}{4} + \frac{4}{3}\right). \tag{18}$$

Plugging this into Equation 16 and solving for y_{sc}, the horizontal displacement of the shear center from the centroid (the vertical location once again being equivalent to that of the centroid due to symmetry), we get

$$y_{sc} = \frac{h^2}{V_z}\left[q_1 + q_2\left(\frac{\pi}{4} + \frac{4}{3}\right) + q_3\right],$$

$$= -3.45\ h. \tag{19}$$

PROBLEM 5.5

We're given a single-cell closed section consisting of three stringers all of area, $A = 0.001$ m^2, and thin webbing of thickness, $t = 0.001$ m. The moment of inertia about the y-axis for the system is given by

$$I_y = (-0.1)^2\ A + (0.1)^2\ A = 2 \times 10^{-5} \text{ m}^4. \tag{20}$$

If the section is subjected to a shear force, $V_z = 5000$ N, then we can determine the total shear flow through the section by adding the base shear component from torsion to the shear resulting from a similar section with a small cut in it. Making a cut between stringers (2) and (3), we can write the resulting shear flows as

$$q'_{21} = \frac{V_z}{I_y}\ ((-0.1)\ A) = -2.5 \times 10^4 \text{ N m}^{-1}, \tag{21}$$

$$q'_{13} = q'_{21} + \frac{V_z}{I_y}\ ((0.1)\ A) = 0, \tag{22}$$

$$q'_{32} = 0. \tag{23}$$

By inspection, V_z acts a distance, $y = -0.267$ m, from the centroid, so we can define the torque acting on the section as

$$T = V_z(-0.267) = -1.33 \times 10^3 \text{ N m}. \tag{24}$$

The average area enclosed by the section can be determined to be

$$\overline{A} = 0.0957 \text{ m}^2. \tag{25}$$

The torque acting on the section gives rise to the base shear flow, q_0, such that

$$T = 2\, \overline{A}\, q_0 \tag{26}$$

We can therefore solve for q_0, giving us

$$q_0 = -6.966 \times 10^3 \text{ N m}^{-1}. \tag{27}$$

Adding this to our cut-section shear flows, we get

$$q_{21} = q'_{21} + q_0 = -3.197 \times 10^4 \text{ N m}^{-1},$$

$$q_{13} = q'_{13} + q_0 = -6.966 \times 10^3 \text{ N m}^{-1},$$

$$q_{32} = q'_{32} + q_0 = -6.966 \times 10^3 \text{ N m}^{-1}.$$

For a shear modulus of $G = 27$ GPa, the twist angle per unit length may be defined as

$$\theta = \frac{1}{2\, G\, \overline{A}} \oint_{\overline{c}} \frac{q}{t}\, ds,$$

$$= \frac{1}{2\, G\, \overline{A}} \left[\int_{-\pi/2}^{\pi/2} \frac{-3.197 \times 10^4}{1.0 \times 10^{-3}}\, (0.1)\, d\theta \right.$$

$$\left. +2 \int_0^{0.81} \frac{-6.966 \times 10^3}{1.0 \times 10^{-3}}\, ds \right],$$

$$= 4.12 \times 10^{-3} \text{ rad} = 0.24°. \tag{28}$$

We can determine the location of the shear center for the cross-section, y_{sc}, through the relation,

$$V_z\, y_{sc} = Q_{21} + (q_{13} + q_{32})\, (0.806)\, (0.0662). \tag{29}$$

where Q_{21} is the component of torque resulting from the shear flow, q_{21}. From the geometry of the section, we can write this as

$$Q_{21} = \int_{-\pi/2}^{\pi/2} q_{21}\, (r + 0.267\, \cos(\theta))\, r\, d\theta,$$

$$= q_{21}\, r\, (\pi\, r + 0.533)\,, \tag{30}$$

where $r = 0.1$ m represents the turning radius of the second web section. In terms of this radius, we can rewrite Q_{21} as

$$Q_{21} = -2.709 \times 10^3 \text{ N}. \tag{31}$$

Plugging this into Equation 29 and solving for y_{sc}, the horizontal displacement of the shear center from the centroid (the vertical location once again being equivalent to that of the centroid due to symmetry), we get

$$y_{sc} = -0.691 \text{ m}. \tag{32}$$

PROBLEM 5.7

We're given a stringer and web section subject to a transverse shear force, $V_z = 5.0 \times 10^3$ N. The section consists of four stringers of area, $A_1 = A_2 = 1.5 \times 10^{-3}$ m and $A_3 = A_4 = 1.0 \times 10^{-3}$ m. The y moment of inertia may be calculated such that

$$I_y = (0.24)^2\, A_1 + (-0.16)^2\, (A_2 + A_3) + (0.04)^2\, A_4,$$

$$= 1.52 \times 10^{-4} \text{ m}^4. \tag{33}$$

Calculating the shear flows through this section gives us

$$q_{43} = \frac{V_z}{I_y}\, (0.04\, A_4) = 1.32 \times 10^3 \text{ N m}^{-1},$$

$$q_{32} = -3.95 \times 10^3 \text{ N m}^{-1},$$

$$q_{21} = -1.18 \times 10^4 \text{ N m}^{-1}.$$

PROBLEM 5.11

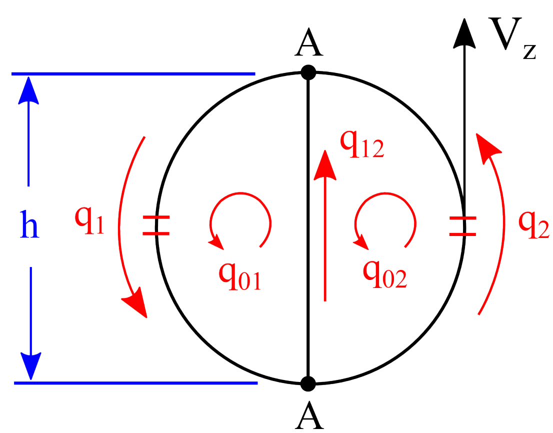

FIG. 3: 5.11 : Thin-walled, two-cell section showing representative side cuts and all flexural shear flow components.

We're given the two-cell, two-stringer, closed section shown in Figure 3, where the height of the section, $h = 0.4$ m, the thickness of the connecting webs is $t = 0.001$ m, and the area of the stringers is $A = 0.001$ m^2. The shear force, V_z, acts a distance, $y = 0.2$ m, from the centroid as shown. The moment of inertia for the section about the y-axis can be written as

$$I_y = (h/2)^2\, A + (-h/2)^2\, A,$$

$$= \frac{1}{2}\, h^2\, A = 8.0 \times 10^{-5} \text{ m}^4. \tag{34}$$

If we cut each cell on the curved outer wall, then $q_1' = q_2' = 0$ and the shear flow, q_{12}', running up the dividing web can be written as

$$q_{12}' = \frac{V_z}{I_y}\left(-\frac{h}{2}\ A\right) = -2.5\ V_z. \tag{35}$$

The torque arising as a result of the applied shear force may be defined as

$$T = 0.2\ V_z. \tag{36}$$

The average area enclosed by each cell is

$$\overline{A} = \frac{\pi}{8}\ h^2 = 0.0628\ \mathrm{m}^2. \tag{37}$$

The applied torque may be defined in terms of the base shear flows through each section, q_{01} and q_{02}, as

$$T = 2\ \overline{A}\ q_{01} + 2\ \overline{A}\ q_{02}. \tag{38}$$

We can determine the twist angle for each section based on their respective base shear flows such that

$$\begin{aligned}\theta_1 &= \frac{1}{2\ G\ \overline{A}} \oint_{\overline{c}_1} \frac{q}{t}\ ds, \\ &= \frac{1}{2\ G\ \overline{A}} \left[\frac{q_{01}}{t}\left(\frac{1}{2}\ \pi\ h\right) + \frac{q_{01} - q_{02}}{t}\ \Big(h\Big)\right], \\ &= \frac{1}{2\ G\ \overline{A}} \left[(1028.32)\ q_{01} - (400)\ q_{02}\right], \\ \theta_2 &= \frac{1}{2\ G\ \overline{A}} \oint_{\overline{c}_2} \frac{q}{t}\ ds, \\ &= \frac{1}{2\ G\ \overline{A}} \left[\frac{q_{02}}{t}\left(\frac{1}{2}\ \pi\ h\right) + \frac{q_{02} - q_{01}}{t}\ \Big(h\Big)\right], \\ &= \frac{1}{2\ G\ \overline{A}} \left[(1028.32)\ q_{02} - (400)\ q_{01}\right].\end{aligned}$$

Geometric compatibility however requires these two angles to be equal. We can therefore derive the relationship,

$$q_{01} = q_{02} \tag{39}$$

Applying this to Equation 38, we can solve for q_{01} and q_{02} such that

$$q_{01} = q_{02} = 0.796\ V_z, \tag{40}$$

where $q_{012} = q_{01} - q_{02} = 0$ represents the base shear flow in the dividing web. Given these base shear flows, we can determine the total shear flows through the section such that

$$q_1 = q_1' + q_{01} = 0.796\ V_z,$$

$$q_2 = q_2' + q_{02} = 0.796\ V_z,$$

$$q_{12} = q_{12}' + q_{012} = -2.5\ V_z.$$

The angle of twist for the section can therefore by determined as

$$\begin{aligned}\theta &= \frac{1}{2\ G\ \overline{A}} \oint_{\overline{c}_1} \frac{q}{t}\ ds, \\ &= \frac{1}{2\ G\ \overline{A}} \left[\frac{q_1}{t}\left(\frac{1}{2}\ \pi\ h\right) + \frac{q_{12}}{t}\ \Big(h\Big)\right], \\ &= -(3.98 \times 10^3)\ \frac{V_z}{G}.\end{aligned} \tag{41}$$

PROBLEM 5.15

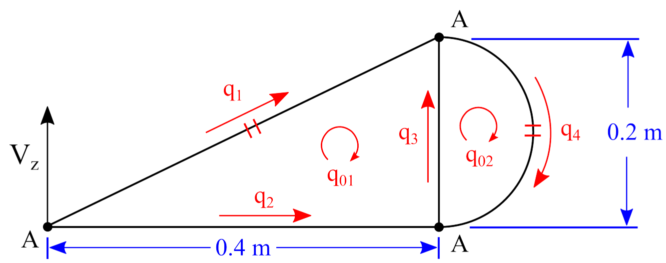

FIG. 4: 5.15 : Thin-walled, two-cell section showing representative side cuts and all flexural shear flow components.

We're given the two-cell, three-stringer, closed section shown in Figure 4, where the thickness of the connecting webs is $t = 0.001$ m and the area of the stringers is $A = 0.001\ \mathrm{m}^2$. The shear force, $V_z = 5000$ N, acts a distance, $y = -0.267$ m, from the centroid as shown. The moment of inertia for the section about the y-axis can be written as

$$I_y = 2\ (-0.0667)^2\ A + (0.1333)^2\ A = 2.67 \times 10^{-5}\ \mathrm{m}^4. \tag{42}$$

If we cut the first cell on the diagonal web and the second cell on the curved web, then we can write the remaining shear flow components as

$$q_2' = \frac{V_z}{I_y}\,(-0.0667\ A) = -1.25 \times 10^4\ \mathrm{N\ m^{-1}},$$

$$q_3' = q_2' + \frac{V_z}{I_y}\,(-0.0667\ A) = -2.50 \times 10^4\ \mathrm{N\ m^{-1}}. \tag{43}$$

The torque arising as a result of the applied shear force may be defined as

$$T = V_z(-0.267) = -1.33 \times 10^3\ \mathrm{N\ m}. \tag{44}$$

The average area enclosed by each cell is

$$\overline{A}_1 = 0.0400\ \mathrm{m}^2, \tag{45}$$

$$\overline{A}_2 = 0.0157\ \mathrm{m}^2. \tag{46}$$

The applied torque may be defined in terms of the base shear flows through each section, q_{01} and q_{02}, as

$$T = 2\ \overline{A}_1\ q_{01} + 2\ \overline{A}_2\ q_{02}. \tag{47}$$

We can determine the twist angle for each section based on their respective base shear flows such that

$$\begin{aligned}\theta_1 &= \frac{1}{2\ G\ \overline{A}_1} \oint_{\overline{c}_1} \frac{q}{t}\ ds, \\ &= \frac{1}{2\ G\ \overline{A}_1} \left[\frac{q_{01}}{t} \left(0.447 + 0.400\right) + \frac{q_{01} - q_{02}}{t} \left(0.200\right) \right], \\ &= \frac{1}{2\ G\ \overline{A}_1} \left[(1047)\ q_{01} - (200)\ q_{02} \right],\end{aligned}$$

$$\begin{aligned}\theta_2 &= \frac{1}{2\ G\ \overline{A}_2} \oint_{\overline{c}_2} \frac{q}{t}\ ds, \\ &= \frac{1}{2\ G\ \overline{A}_2} \left[\frac{q_{02}}{t} \left(\pi\ (0.1)\right) + \frac{q_{02} - q_{01}}{t} \left(0.2\right) \right], \\ &= \frac{1}{2\ G\ \overline{A}_2} \left[(514.16)\ q_{02} - (200)\ q_{01} \right].\end{aligned}$$

Geometric compatibility however requires these two angles to be equal. We can therefore derive the relationship,

$$(3.89 \times 10^4)\ q_{01} = (3.77 \times 10^4)\ q_{02}. \tag{48}$$

Applying this to Equation 47, we can solve for q_{01} and q_{02} such that

$$q_{01} = 1.187 \times 10^4\ \text{N m}^{-1},$$

$$q_{02} = 1.223 \times 10^4\ \text{N m}^{-1}.$$

Given these base shear flows, we can determine the total shear flows through the section such that

$$q_1 = q_1' + q_{01} = 1.187 \times 10^4\ \text{N m}^{-1},$$

$$q_2 = q_2' + q_{01} = -6.343 \times 10^2\ \text{N m}^{-1},$$

$$q_3 = q_3' + q_{01} - q_{02} = -2.537 \times 10^4\ \text{N m}^{-1},$$

$$q_4 = q_4' + q_{02} = 1.223 \times 10^4\ \text{N m}^{-1}.$$

Given these shear flows, we can determine the location of the shear center for the cross-section, y_{sc}, through the relation,

$$\begin{aligned}V_z\ y_{sc} = q_1\ (0.447)\ (0.0596) + q_2\ (0.4)\ (0.0667) \\ + q_3\ (0.2)\ (0.133) + Q_4,\end{aligned} \tag{49}$$

where Q_4 is the component of torque resulting from the shear flow, q_4. From the geometry of the section, we can write this as

$$\begin{aligned}Q_4 &= \int_{-\pi/2}^{\pi/2} q_4\left(r + 0.133\ \cos(\theta)\right) r\ d\theta, \\ &= q_4\ r\left(\pi\ r + 0.267\right),\end{aligned} \tag{50}$$

where $r = 0.1$ m represents the turning radius of the second web section. In terms of this radius, we can rewrite Q_4 as

$$Q_4 = 710.35\ \text{N m}. \tag{51}$$

Plugging this into Equation 50 and solving for y_{sc}, the horizontal displacement of the shear center from the centroid (the vertical location once again being equivalent to that of the centroid due to symmetry), we get

$$y_{sc} = 0.0666\ \text{m}. \tag{52}$$